Pergamon

Int. J. Heat Mass Transfer. Vol. 41, No. 14, pp. 2125–2130, 1998

Printed in Great Britain
0017-9310/98 $19.00+0.00

PII: S0017-9310(97)00350-5

The mean temperature field of a buoyancy-induced boundary layer adjacent to a vertical plate immersed in a stratified medium

JOSEF TANNY
Center of Technological Education Holon, P.O.B. 305, Holon 58102, Israel

and

JACOB COHEN†
Faculty of Aerospace Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel

(*Received* 3 *June* 1997 *and in final form* 7 *November* 1997)

Abstract—The mean temperature field of a buoyancy-induced boundary layer adjacent to an isothermal vertical flat plate immersed in a linear ambient thermal stratification is investigated experimentally. Experiments are carried out in an apparatus, consisting of a square perspex aquarium and a heated plate assembly. Mean temperature measurements are conducted using constant current fiber-film temperature sensors. The temperature profiles for unstratified and stratified media are compared with corresponding theoretical similarity solutions and good agreement is obtained. Accordingly, for the case in which the environment is thermally stratified, the mean temperature profiles possess a region of temperature deficit (negative dimensionless temperature) and the thickness of the boundary layer is approximately independent of the streamwise coordinate. Measurements of the local heat transfer are in good agreement with corresponding theoretical predictions. © 1998 Elsevier Science Ltd. All rights reserved.

1. INTRODUCTION

Buoyancy-induced flows frequently occur around bodies surrounded by a thermally stratified environment which may influence considerably the flow and the heat transfer. This effect is of particular importance in energy storage systems such as solar ponds or in heat transfer from bodies in enclosures where the thermal input itself may produce the stratification.

This work is focused on free convection flow adjacent to the surface of a heated flat vertical plate immersed in a thermally stratified environment. Previous studies for various boundary conditions and ambient temperature distributions (for a comprehensive review see Gebhart *et al.* [1]) have shown that stratification increases the local heat transfer coefficient and decreases the velocity and buoyancy levels. Another considerable effect of the stratification on the mean field is the formation of a region with a temperature deficit (i.e. a negative dimensionless temperature) and flow reversal in the outer part of the boundary layer. This phenomenon was first shown theoretically by Prandtl [2] for an infinite wall and later on by Jaluria and Himasekhar [3] for semi-infinite walls, utilizing finite difference techniques. More recently, boundary layer similarity solutions were found by Kulkarni, Jacobs and Hwang [4] and by Henkes and Hoogendoorn [5].

Only few experimental studies were carried out on vertical free convection in a stratified environment. Jaluria and Gebhart [6] studied the stability of the flow adjacent to a vertical plate dissipating a uniform heat flux into a stratified medium both theoretically and experimentally. For this case a theoretical similarity solution exists, in which the ambient stratification varies like $X^{1/5}$, where X is the downstream coordinate. Unlike the case of linear stratification [4, 5], the flow reversal and temperature deficit in this case [6], where the variation of the ambient temperature is relatively weak, are extremely small.

The heat transfer from an isothermal vertical surface to a thermally stratified environment was studied theoretically and experimentally by Chen and Eichhorn [7]. Their results confirmed the expected increase of the heat transfer coefficient with the stratification level. In the above studies no measured profiles of mean temperature (or velocity) field were reported.

The results presented here are the first experimental part of a more comprehensive study, the main goal of which is to understand the effect of ambient stratification on the instability mechanisms of free convection flows. In a previous theoretical paper, Krizhevsky, Cohen and Tanny [8] studied the stability characteristics of a buoyancy induced flow adjacent to an isothermal vertical plate immersed in a linear

† Author to whom correspondence should be addressed.

NOMENCLATURE

D	length of the heated plate
g	gravitational acceleration
Gr	Grashof number $= g\beta(T_w - T_\infty(0))l^3/\nu^2$
h	local heat transfer coefficient
H	similarity temperature function $= (T - T_\infty)/(T_w - T_\infty)$
k	thermal conductivity
l	length scale $= \sqrt[4]{\nu^2/(g\beta(\mathrm{d}T_\infty/\mathrm{d}X))}$
L	downstream position where $T_\infty(L) = T_w$
m	$= D/L$
Nu	local Nusselt number $= h(X)X/k$
$\overline{Nu}$	average Nusselt number
Pr	Prandtl number $= \nu/\kappa$
T	temperature
X	downstream coordinate
Y	transverse (normal to the wall) coordinate
$Y_{0.5}$	thickness of thermal boundary layer.

Greek symbols

β	coefficient of thermal expansion
η	similarity variable
κ	thermal diffusivity
ν	kinematic viscosity.

Subscripts

s	stratified
u	unstratified
w	wall
∞	ambient.

ambient thermal stratification. It was found that because of the flow reversal, induced by the ambient stratification, this flow is susceptible to absolute instability. To confirm this finding experimentally, we first investigate here the characteristics of the mean flow before the onset of instability. Thus, the purpose of the present work is to study experimentally the mean temperature field and the heat transfer of this flow and to compare it with existing laminar similarity solutions.

2. EXPERIMENTAL SETUP AND PROCEDURES

The experiments were carried out in a plexiglass aquarium, 50 × 50 × 50 cm high, filled with filtered tap water (Fig. 1). The heated plate assembly was made of a flat rectangular box with dimensions of 17 × 10 × 3 cm through which water at a predetermined temperature could circulate. One side of the box was made of a 1.5 mm thick aluminum plate covered with a thin brass foil (0.3 mm) and served as the heating surface. The other sides of the box were made of an insulating plastic material. A sharp leading edge was attached to the bottom part of the box. Constant-temperature water was provided by a HAAKE F3-K circulator.

The ambient thermal stratification was obtained by filling the tank using the 'double-bucket' method [9]. This method produced a linear temperature profile and allowed us to approximately control the desired temperature gradient across the tank. After the linear temperature profile was established, it was maintained by heating and cooling the top and the bottom of the tank, respectively, to the prescribed temperatures. The top temperature was controlled by means of an electrical heating element which was in contact with the water surface. The bottom temperature was controlled by placing the test tank in a wider shallow water-cooled basin in which water from another constant-temperature bath could circulate (see Fig. 1). The plexiglass bottom of the tank was in contact with the circulating water. The sidewalls of the tank were insulated by means of transparent thermal insulation, consisting of plexiglass plates separated by thin air gaps. In this way a linear temperature profile could be attained within a relatively short time (about 2 h) and then maintained for a long period of time.

During the filling process the ambient stratification in the tank was measured continuously by a fixed vertical rake of 19 thermocouples type *T*. The thermocouples were separated by a uniform vertical distance of 2.5 cm. Their output was measured, linearized and recorded by a data-acquisition system consisting of an A/D card, a multiplexer and a 486 PC. This measurement was also carried out during the experiment to provide a quick reference of the instantaneous ambient stratification.

The temperature profiles across the boundary layer were measured by a fiber-film sensor (Dantec 55R11) consisting of a 70 μm quartz fiber with a 2 μm quartz coating. The sensor was operated by a constant-current anemometer. The probe was traversed in the tank by a computerized three-dimensional traversing mechanism with a minimum linear move of 5 μm. The output of the fiber sensor was recorded by a separate data-acquisition system consisting of an A/D card and a 486 PC. Prior to each experiment, the fiber sensor was calibrated using a constant temperature circulator (HAAKE F3-K). The calibration curve was linear over the temperature range considered. The typical sensitivity of the sensor was relatively high of about 2.4°C/V. Such a sensitivity allowed the identification of very small temperature differences, which is usually the case in free convection flows.

After calibration, the sensor was located at the level

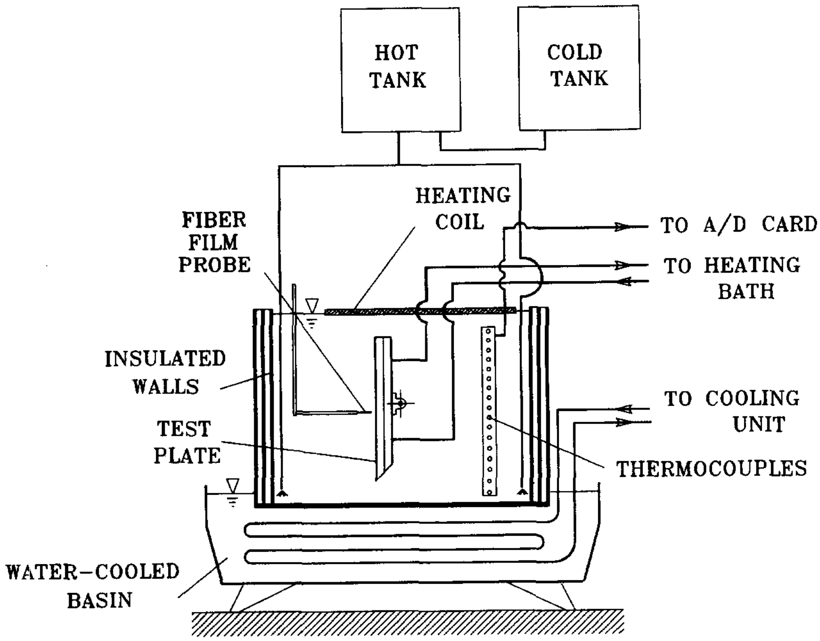

Fig. 1. A schematic view of the experimental apparatus.

of the leading edge of the heated plate. This position was the reference level denoted as $X = 0$. From this level the sensor was traversed only by the computerized system such that its vertical position was always known relative to $X = 0$. Before each experiment the ambient thermal stratification was measured by the sensor at a distance of about 1 cm from the plate.

To start an experiment the sensor was moved to the desired X position and was located at its first station, in contact with the plate. The water circulator was turned on and the temperature rise was detected by the sensor. As the temperature reached a constant value (after about 1 min), a steady state was assumed and the profile measurement started. This was done by moving the sensor away from the plate in predetermined 50 uniform steps of 0.175 mm using the computerized traversing system. After each profile measurement the water circulation was turned off for a period of about 60 min during which all motions decayed and the fluid temperature near the plate was equalized to the ambient temperature. To study the heat transfer, profiles of seven stations only (instead of 50) were measured near the wall, from which the local temperature gradient at the wall was estimated using least square fit.

3. RESULTS AND DISCUSSION

In the profiles shown below the nondimensional temperature is defined as: $(T-T_\infty(0))/(T_w-T_\infty(0))$ where T_w is the uniform wall temperature and $T_\infty(0)$ is the ambient temperature at the height of the leading edge ($X = 0$). However, the temperature measurement at the wall and at very small distances from it was found to be inaccurate due to the 'flow-blocking' phenomenon occurring when the sensor is extremely close to the wall. Therefore, in each profile, the first 1–2 data points were discarded and, in accordance with the similarity solution, the next 3–4 points were used to linearly extrapolate the temperature profile up to the first station at the wall. By this method the local wall temperature T_w and the temperature gradient at the wall were estimated and used in the results below. The normal distance from the wall, Y, was normalized with the thickness of the thermal boundary layer $Y_{0.5}$, which was defined as the distance at which $(T-T_\infty(0))/(T_w-T_\infty(0)) = 0.5$.

As a reference, the cross stream mean temperature profile was measured for the case in which the ambient temperature was uniform. A typical example of a normalized mean temperature profile measured in this case is shown in Fig. 2, where T_∞ is the ambient uniform temperature. As can be seen the measurements (symbols) conform to the self-similar solution (solid line) [1], indicating that we are able to obtain, experimentally, a laminar buoyant boundary layer.

To study the temperature field in a stratified medium, experiments were conducted for different ambient stratifications in the range of 0.26–0.57°C/cm. As is shown in Fig. 3, the three different experimental ambient temperature profiles (symbols) are fitted very well by linear curves (solid lines).

The nondimensional parameters governing this flow are the Grashof and Prandtl numbers. The Grashof number is defined as:

$$Gr = g\beta(T_w - T_\infty(0))l^3/\nu^2 \tag{1}$$

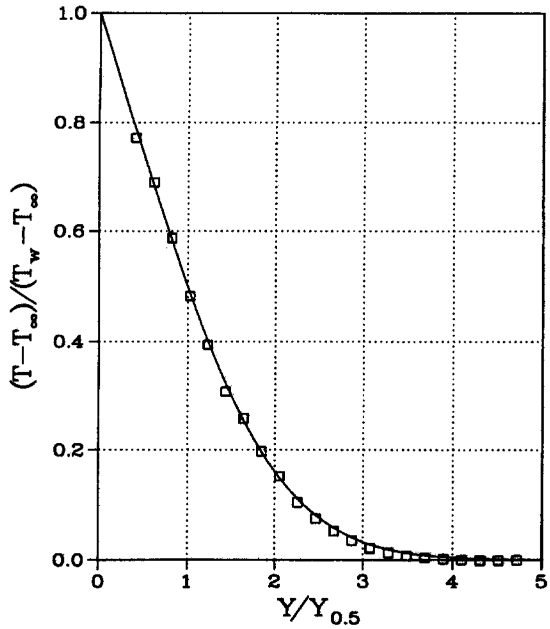

Fig. 2. The mean temperature profile measured in an unstratified medium. The symbols represent the measured data and the solid line is the corresponding theoretical solution. $X = 10$ cm, $T_w = 46.4$°C and $T_\infty = 26.9$°C.

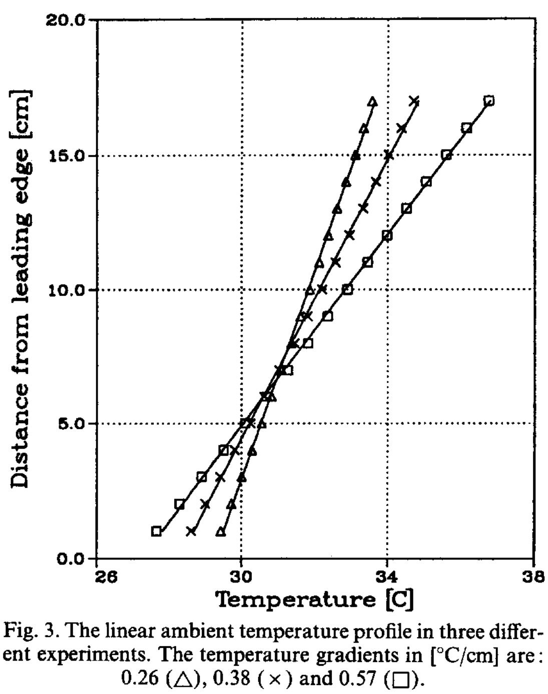

Fig. 3. The linear ambient temperature profile in three different experiments. The temperature gradients in [°C/cm] are: 0.26 (△), 0.38 (×) and 0.57 (□).

where g is the acceleration due to gravity, β is the coefficient of thermal expansion, ν is the kinematic viscosity and l is the length scale:

$$l = \sqrt[4]{\nu^2/g\beta(\mathrm{d}T_\infty/\mathrm{d}X)}. \quad (2)$$

The physical properties are represented by the Prandtl number, $Pr = \nu/\kappa$ where κ is the thermal diffusivity.

To assure laminar flow, the experiments were carried out at relatively low Gr numbers. Combined with technical limitations, this facilitated experiments only in a narrow range of Gr. In each experiment temperature profiles were measured at several downstream locations. In Fig. 4(a), five profiles are shown for the case in which the ambient temperature gradient was relatively high (0.57 [°C/cm] and $Gr = 125$). The symbols represent the measured data points and the solid line is the corresponding theoretical solution [4], which was calculated [8] for $Pr = 4.8$. This Prandtl number corresponds to the average film temperature associated with the measured profiles. In Fig. 4(b), which follows the same structure as that of Fig. 4(a), temperature profiles are presented for the case in which the ambient temperature gradient was relatively low (0.31 [°C/cm] and $Gr = 133$) and the theoretical profile was calculated at $Pr = 5.1$. As can be seen, for both ambient stratifications, the profiles possess a region of temperature deficit and the agreement between the theoretical mean profile and the measured ones is fairly good. The slight deviation in the outer region may be due to the variation of Pr with temperature in the ambient stratified fluid which is not accounted for in the theory.

According to the similarity solution [4, 8], the thickness of the thermal boundary layer, $Y_{0.5}$, is independent of the downstream coordinate. We calculated the theoretical $Y_{0.5}$ for each experiment and compared it with the thickness measured from the above profiles. For the experiment presented in Fig. 4(a), the theoretical thickness is 0.0867 cm while the average value of the thickness measured for the five profiles is 0.0886 cm. The maximum deviation between the theoretical and the measured values is 12%. In the second experiment (Fig. 4(b)) the theoretical thickness is 0.105 cm, the average measured one is 0.088 cm, and the maximum deviation is 19%. A possible reason for these deviations is that in the experiments the wall temperature was not exactly uniform, usually increasing in the downstream direction. Over the tested region, the wall temperature was uniform within ±0.4°C (Fig. 4(a)) and ±0.6°C (Fig. 4(b)).

To examine the effect of the ambient stratification on the heat transfer, the local Nusselt number,

$$Nu = h(X)X/k = \frac{X\,\mathrm{d}T/\mathrm{d}Y(0)}{T_w - T_\infty}, \quad (3)$$

was compared with the one measured in the case of an unstratified medium. For this comparison to be meaningful, the average temperature difference between the wall and the ambient fluid over the entire plate in the stratified case was made equal to the constant temperature difference in the unstratified experiment. This comparison is presented in Fig. 5. The triangular symbols correspond to the experimental data obtained when the ambient stratification

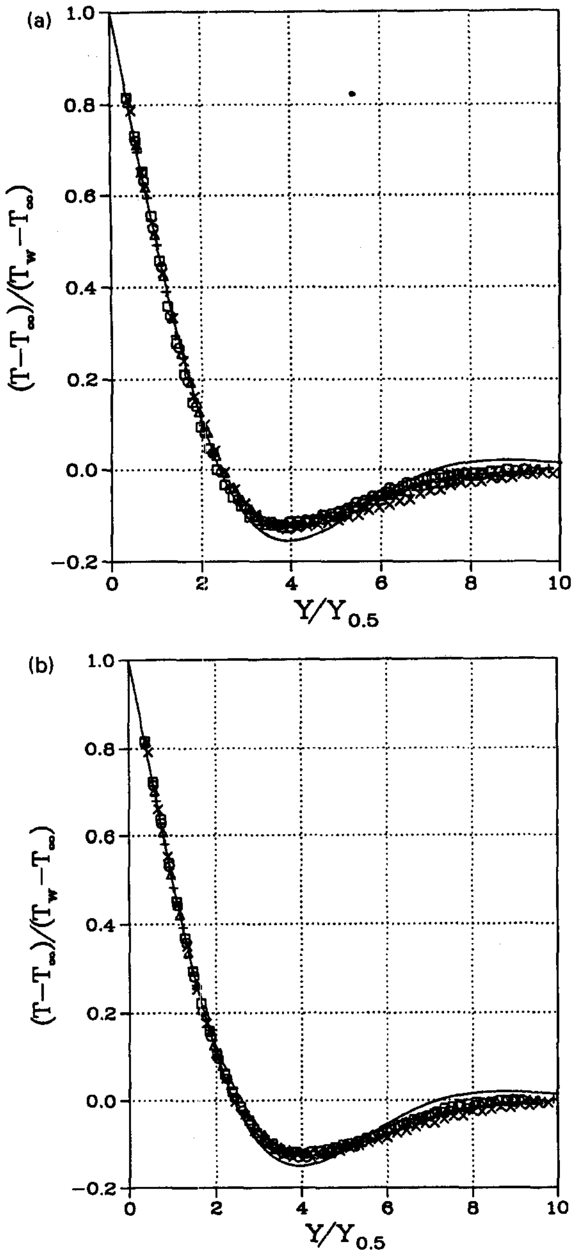

Fig. 4. The mean temperature profile measured in a stratified medium. The symbols represent the measured data and the solid lines are the corresponding theoretical solutions for (a) $Pr = 4.8$ and (b) $Pr = 5.1$. (a) Temperature gradient is 0.57 [°C/cm], $Gr = 125$, $X = 10$ cm (×), $X = 12$ cm (+), $X = 13$ cm (△), $X = 14$ cm (○) and $X = 15$ cm (□). (b) Temperature gradient is 0.31 [°C/cm], $Gr = 133$, $X = 8$ cm (×), $X = 10$ cm (+), $X = 12$ cm (△), $X = 14$ cm (○) and $X = 15$ cm (□).

was 0.42 [°C/cm], while the square symbols correspond to an experiment in which the environment was unstratified.

The respective theoretical predictions [1, 4, 8] were obtained by:

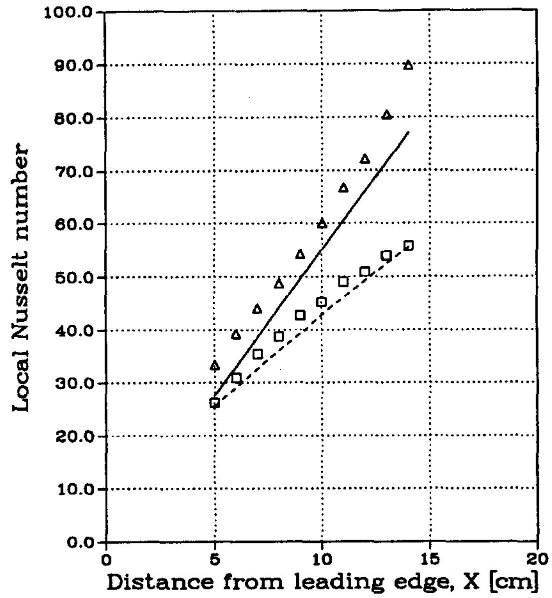

Fig. 5. The downstream variation of the local Nusselt number. Unstratified medium: □, experimental data for $T_w = 32.6$°C and $T_\infty = 29.55$°C; dashed line, theoretical prediction for $Pr = 5.26$. Stratified medium: △, experimental data for $T_w = 32.6$°C, $T_\infty(0) = 25.95$°C and $dT_\infty/dX = 0.42$°C/cm; solid line, theoretical prediction for $Pr = 5.26$.

$$Nu_s(X) = \frac{-H_s'(0)X}{\sqrt{2}}\left[\frac{g\beta(dT_\infty/dX)}{\nu^2}\right]^{1/4} \tag{4}$$

for the stratified case (solid line), and by

$$Nu_u(X) = \frac{-H_u'(0)}{\sqrt{2}}\left[\frac{g\beta(T_w - T_\infty)X^3}{\nu^2}\right]^{1/4} \tag{5}$$

for the unstratified case (dashed line), where the prime symbol denotes differentiation with respect to the similarity variable η. The agreement between the experimental data and the theoretical results is fairly good, which supports our use of the theoretical predictions in the following, to approximately estimate the heat transfer coefficient. Again, the deviations between the theoretical and experimental results may be due to the variation of Pr in the ambient stratified fluid.

It should be noted that as the stratification approaches zero, $Nu_s \to 0$, instead of reducing to the unstratified limit. This is a property of the similarity solution of Kulkarni *et al.* [4]. According to this solution, the ambient temperature stratification is given by

$$T_w - T_\infty(X) = (T_w - T_\infty(0))\left(1 - \frac{X}{L}\right) \tag{6}$$

where L is the downstream position at which $T_\infty(L) = T_w$ or, equivalently,

$$L = (T_w - T_\infty(0))/(dT_\infty/dX). \tag{7}$$

It is assumed that L is larger than the length of the plate D, such that the wall temperature is above that of the surrounding fluid at all locations. In the limit $\mathrm{d}T_\infty/\mathrm{d}X \to 0$, the boundary layer thickness l becomes infinite (equation (2)) and this is the reason why the heat transfer coefficient approaches zero.

A parameter of interest is the ratio between the local Nusselt numbers:

$$\frac{Nu_s(X)}{Nu_u(X)} = \frac{H_s'(0)}{H_u'(0)}\left[\frac{X(\mathrm{d}T_\infty/\mathrm{d}X)}{(T_w - T_{\infty u})}\right]^{1/4}. \quad (8)$$

The effect of the stratification on this ratio is through the parameter within the square brackets, and is qualitatively similar to the effect reported by Chen and Eichhorn [7].

Substitution of equation (7) into equation (8) and using the fact that the average temperature difference between the wall and the ambient fluid over the entire plate in the stratified case is equal to the constant temperature difference in the unstratified case, leads to

$$\frac{Nu_s(X)}{Nu_u(X)} = \frac{H_s'(0)}{H_u'(0)}\left[2\frac{X}{L}\right]^{1/4}. \quad (9)$$

This implies that the ratio between the local convective heat transfer coefficients increases with the downstream distance (as shown in Fig. 5). The ratio between the average heat transfer coefficients [obtained by integrating equation (9)] is

$$\frac{\overline{Nu_s(X)}}{\overline{Nu_u(X)}} = 0.8 \times (2m)^{1/4}\frac{H_s'(0)}{H_u'(0)} \quad (10)$$

where $m = D/L \leqslant 1$. Thus, this ratio can be greater or less than one, depending on the Prandtl number $(H_s'(0)/H_u'(0))$ and the stratification level (m).

4. CONCLUDING REMARKS

Mean temperature profiles of a buoyancy-induced boundary layer adjacent to an isothermal vertical flat plate immersed in a linear ambient thermal stratification were measured. Comparison between these profiles and self-similar solutions shows good agreement. Unlike the unstratified case, the mean temperature profiles possess a region of temperature deficit and the thickness of the boundary layer is approximately independent of the streamwise coordinate. The local Nusselt numbers were measured for stratified and unstratified media. Comparison between the measured values and those predicted theoretically shows good agreement. Using existing theoretical similarity solutions, the ratio between the average heat transfer coefficients for the stratified and unstratified media was derived. This ratio was found to be larger or smaller than 1, depending on the Prandtl number and the stratification level.

Acknowledgements—The authors would like to thank Eli Naimark for his invaluable assistance in modifying the experimental apparatus and conducting the experiments, and Amos Beer for constructing the apparatus. We also thank Edwin Malkiel for his help with the data acquisition software. This work was supported by the Israel Science Foundation under Grant no. 522/92.

REFERENCES

1. Gebhart, B., Jaluria, Y., Mahajan, R. and Sammakia, B., *Buoyancy Induced Flows and Transport*. Hemisphere publishing co., 1988.
2. Prandtl, L., *Essentials of Fluid Dynamics*. Blackie, London, 1952, pp. 422–425.
3. Jaluria, Y. and Himasekhar, K., Buoyancy induced two dimensional vertical flows in a thermally stratified environment. *Computer and Fluids*, 1983, **11**, 39–49.
4. Kulkarni, A. K., Jacobs, H. R. and Hwang, J. J., Similarity solution for natural convection flow over an isothermal vertical wall immersed in thermally stratified medium. *International Journal of Heat and Mass Transfer*, 1987, **30**(4), 691–698.
5. Henkes, R. A. W. M. and Hoogendoorn, C. J., Laminar natural convection boundary layer flow along a heated vertical plate in a stratified environment. *International Journal of Heat and Mass Transfer*, 1989, **32**, 147–155.
6. Jaluria, Y. and Gebhart, B., Stability and transition of buoyancy induced flows in a stratified medium. *Journal of Fluid Mechanics*, 1974, **66**, 593–612.
7. Chen, C. C. and Eichhorn, R., Natural convection from a vertical surface to a thermally stratified fluid. *Journal of Heat Transfer*, 1976, 446–451.
8. Krizhevsky, L., Cohen, J. and Tanny, J., Convective and absolute instabilities of a buoyancy-induced flow in a thermally stratified medium. *Physics of Fluids*, 1996, **8**(4), 971–977.
9. Oster, G., Density gradients. *Scientific American*, 1965, **213**, 70–76.